

**MATH 3705\* A    Test 1    January 2012**

**LAST NAME:** \_\_\_\_\_ **ID#:** \_\_\_\_\_

Questions 1-6 are multiple choice. Circle the correct answer. Only the answer will be marked.

1. [2]  $\mathcal{L}\{e^{-2t} \cos(3t)\} =$

(a)  $\frac{s+2}{(s+2)^2+9}$       (b)  $\frac{s}{s^2+9}$       (c)  $\frac{s-2}{(s-2)^2+9}$       (d) None of the above

2. [2]  $\mathcal{L}\{t \sin(3t)\} =$

(a)  $\frac{6s}{(s^2+9)^2}$       (b)  $\frac{s^2-9}{(s^2+9)^2}$       (c)  $\frac{9-s^2}{(s^2+9)^2}$       (d) None of the above

3. [2]  $\mathcal{L}\{u(t-1)e^{2t}\} =$

(a)  $\frac{e^{-s}}{s-2}$       (b)  $\frac{e^{1-s}}{s+2}$       (c)  $\frac{e^{2-s}}{s-2}$       (d) None of the above

4. [3]  $\mathcal{L}^{-1}\left\{\frac{2s}{s^2-25}\right\} =$

(a)  $e^{5t} - e^{-5t}$       (b)  $e^{5t} + e^{-5t}$       (c)  $-\cos(5t)$       (d) None of the above

5. [3]  $\mathcal{L}^{-1}\left\{\frac{se^{-3s}}{s^2+4}\right\} =$

(a)  $u(t-3) \cos(2t)$       (b)  $u(t+3) \cos[2(t+3)]$

(c)  $u(t-3) \cos[2(t-3)]$       (d) None of the above

6. [3]  $\mathcal{L}^{-1}\left\{\frac{3s+6}{s^2-4s+13}\right\} =$

(a)  $3e^{2t} \cos(3t) + 4e^{2t} \sin(3t)$       (b)  $3e^{-2t} \cos(3t) + \frac{9}{4}e^{-2t} \sin(3t)$

(c)  $3e^{2t} \cos(3t) + \frac{9}{4}e^{2t} \sin(3t)$       (d) None of the above

**Answers:** 1.(a), 2.(a), 3.(c), 4.(b), 5.(c), 6.(a).

7. [5 marks] Let  $f(t) = e^t$  for  $0 < t < 1$  and  $f(t+1) = f(t)$  for all  $t \geq 0$ . Find  $\mathcal{L}\{f(t)\}$ .

Solution:

Since  $f$  is periodic with the period  $\omega = 1$ , then

$$\begin{aligned}\mathcal{L}\{f(t)\} &= \frac{1}{1 - e^{-s}} \int_0^1 e^t e^{-st} dt = \frac{1}{1 - e^{-s}} \int_0^1 e^{t(1-s)} dt = \frac{1}{1 - e^{-s}} \left\{ \frac{1}{1-s} e^{t(1-s)} \right\}_0^1 = \\ &= \frac{1}{1 - e^{-s}} \left( \frac{1}{1-s} \right) (e^{1-s} - e^0) = \frac{e^{1-s} - 1}{(1 - e^{-s})(1 - s)}.\end{aligned}$$

8. [10 marks] Employ the Laplace transform to solve the initial-value problem

$$y'' + 4y' + 8y = 0, \quad y(0) = 2, \quad y'(0) = 4.$$

Solution:

$$[s^2 Y(s) - sy(0) - y'(0)] + 4[sY(s) - y(0)] + 8Y(s) = 0$$

$$\Rightarrow (s^2 + 4s + 8)Y(s) - 2s - 12 = 0 \Rightarrow Y(s) = 2 \frac{s + 6}{s^2 + 4s + 8} = 2 \frac{(s + 2) + 4}{(s + 2)^2 + 4} =$$

$$2 \frac{s + 2}{(s + 2)^2 + 4} + 4 \frac{2}{(s + 2)^2 + 4} \Rightarrow y(t) = 2e^{-2t} \cos(2t) + 4e^{-2t} \sin(2t).$$

**Marking Guidelines:** Give 5 marks for the correct direct transformation and 5 marks for the correct inverse. Take 1 mark off for a minor arithmetic mistake.